Inventory Cost Optimization with Normally Distributed Lead Time and Stochastic Demand Considering Fill Rate

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Abstract—In this research, inventory stochastic model has been used to find out the optimum re-order point and ordering quantity. It is one of the important ways to find the total inventory cost. A hybrid equation of total inventory cost has been developed where customer demand has been kept deterministic up-to re-order point and after re-order point, it has become probabilistic. Demand has been considered as a random variable after re-order point. Some equations have been derived to find the ordering quantity Q and re-order quantity R. In the traditional EOQ model, holding cost is the main portion of the cost which is considered. But in this hybrid model, a penalty cost is included which has a clear relationship with fill rate. So, some equations have also been derived to link up the total cost with the re-order quantity, ordering quantity and penalty cost in this research. Besides, fill rate has also been measured in terms of those ordering quantity and reorder quantity.

Index Terms— Demand Rate, Lead Time, Ordered Quantity, Re-order quantity. Shortage/Penalty Cost, Holding Cost, Demand Elasticity

1 Introduction

INVENTORY is one of the most important parts of the supply chain. Supply chain costs are much more dependent on inventory cost. Inventory cost has three main components- Holding cost, procurement cost, and the shortage cost. During demand uncertainty, the service level is much more important [1]. In an easy word, it can be said that if the inventory goes stock out or not to fulfill the required customer demand. If demand is more than the inventory, then stockout occurs [2].

This article is concerned with an inventory model which is quite simple and standard. Attention is restricted to a simple familiar class of control policies the re-order/order quantity or (Q, R) policies. When the inventory position reaches the reorder quantity R, an order is placed for the fixed amount Q, the batch size. Normally demand rate is assumed to be a given constant but here demand is constant up to reorder point and then the demand is uncertain. The change in demand in response to inventory or marketing decisions is commonly referred to as demand uncertainty [2].

Recently many researchers have developed a series of theories to find the optimal inventory cost such that base stock (Q, R) model.

As the demand is probabilistic, lead-time demand plays an important role in inventory management. If demand is unusually large, a stock out may occur or emergency actions may be required to avoid a stock out. On the other hand, if demand is lower than anticipated, the replenishment arrives earlier than needed and inventory is carried. Managers have a different perspective on how to balance these two types of risks [3]. The aim of this article is to determine the optimal reorder quantity R and the quantity ordered Q for the (Q, R) inventory system.

2 BACKGROUND

A supply chain consists of all parties involved, directly or indirectly to fulfill customer's request or demand. Only manufacturers and suppliers are not the only element of a supply chain. It also includes transporters, warehouses, retailers and customers too. Every organization wants to maintain a good supply chain policy for their customer satisfaction [5]. Customer is normally known as the end users. But in a supply chain, customer can be the retailer; it can be the warehouse or local supplier. Customers play the most significant part in business. In fact the customer is the actual boss in a deal and is responsible for the actually profit for the organization. Customer is the one who uses the products and services and judges the quality of those products and services. Hence it's important for an organization to retain customers or make new customers and flourish business. Kumar and Sharman observed that reliable delivery is 2nd in rank where product attributes are 1st in case of customer satisfaction [6]. But fill rate is most important parameter that described the average percentage of products that are shipped from the stock [4]. Johnson et al. showed that the traditional expression for the line fill rate which performs well for high fill rate which is above 90%.

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But the problem is they continuously underrated the authentic fill rate. They developed an exact fill rate expression which was emphatic for the both case of high and low fill rate. An alternative fill rate expression was derived, which was cogent for the case of higher fill rate [7].

Operational managers always try to control the shortage incidents. Product extension is one of the major challenges to the operational managers. It is difficult to forecast the demand if the product is highly proliferated[8]. Different product versions are often developed for different segmentations of merchant. Effective benefits can be achieved by proper investigation of the opportunities in the design of the product to achieve controlled service level [9]. Service level is used in supply chain management and in inventory management to measure the performance of inventory replenishment policies. The probability of arriving customer order was called the cycle service level. Penalty cost per backordering system for SKU was considered [10]. Under consideration, from the optimal solution of such a model also the optimal size of back orders can be derived. Service level can be said as the probability of satisfying customer order from the inventory. That means it is the probability that the inventory goes stock-out or not to fulfill the demand. Besides, customer can be satisfied by one of the several items. Customer service can be influenced by the substitution of demand. A probabilistic demand model and methodology was selected, which helped to maximize the profit subjected to resource constraints [11].

Voudouris presented a mixed integer linear programming model that improved the scheduling process by avoiding the material stock out of resource violation for a formulation and packaging chemical plant. Five types of constraints were subjected in that research. ABC inventory classification is widely used with customers demand value. Service level is strongly related with stock keeping units [12]. A multi objective supply chain model was developed by Sabri and Beamon where described that service levels and fill rates are the performance measurement system of supply chain [13]. Demand uncertainty is a vector of total supply chain network, was shown in this paper.

Teunter, Babai and Syntetos studied that the cost criterion is best in case of fixed cycle service level instead of fixed fill rates [14]. Configuration capacity, inventory level and complexity on service performance have indicative impact on service performance which is measured by order fill rates in a configure-to-order environment [15]. The result of this study suggests about differential direct and interactive effects of examined variables on order fill rates. Fill rate is the fraction of demand that is directly filled from the stock on hand [16]. The fill rate is the fraction of customer demand that is met through immediate stock availability, without backorders or lost sales. Simply, it can be said that fill rate is the total number of units that are actually filled based on the total that is ordered. It is the difference between how much is ordered versus how much is filled. Service level is related with the penalty cost for shortage [17]. This can be used to calculate the base stock amount for the minimum carrying cost [18].

Inventory is the stock of materials or finished goods a manufacturer or seller keeps to cater to fluctuations in unanticipated demand from the consumer end. When the supplier runs out of the particular product in demand occurring within a definite lead time, a stock-out of the product occurs and has to incur penalty cost of lost sales. This is what inventory cost signifies and this cost is known as stock-out/shortage cost. When stock outs are backordered, average outstanding backorders are become the key performance criteria. The outstanding backorders are denoted by B which is function of q & r. Order size is the multiple of q, which is chosen to rise up the inventory position to "I" according to the study of Richards [20] and Zipkin [21]. In this paper, it is proved that poor performance of service level occurred in case of two service levels. Exact expression is needed for the convex function in practical work. Approximation is reasonable in terms of discrete demands. Zipkin shows that the backorders are the important measure service and it is the component to express average inventory [21].

If product is not available on shelf, supermarkets will lose their revenue. Gruen et al. reported the percentages of stock out which are 8.3% in average, 8.6% in Europe. US faces the percentage of 7.9% [22]. Retail competition in supply chain management achieves strategic importance by good assortment, shelf availability and food supply [23]. Food retail inventory management is highly depended on high customer level and uncertain demand [24]. In this paper, limited shelf life, positive lead time, LIFO or FIFO issuing policy and multiple service level constraints were considered to present a method of determining the order quantities of perishable products. Their research focused on the inventory problems related with positive lead time, lost sales and service level constraints.

3 Methodology of the Proposed Research

In order to carry out this research work, steps that have been adopted are mentioned below:

1. Getting suitable knowledge on inventory by reading different article, journal papers and thesis papers.
2. Inventory cost optimization techniques were selected by determining optimum order quantity (Q) and optimum re-order quantity (R).
3. One objective function was developed.
4. Ordering costs, holding costs, shortage costs (if any) were determined.
5. Amount of shortage was calculated by using Leibnitz theorem.
6. When shortages occur there had been a penalty cost which was multiplied by the amount of shortage to calculate the penalty cost.
7. Finally, total cost was found by adding those three costs.
8. Parameters were found from total cost equation.
9. Total cost was differentiated with respect to order quantity (Q) and re-order point (R) respectively.
10. Sensitivity analysis was done by using MATLAB to find out greater sensitive parameter.
11. By using MATLAB some graphs were drawn to understand the sensitive parameters which make a greater impact on total cost.

Moreover, this research is based on real-life inventory data. Inventory systems were tried to be observed carefully. But everything has some limitations. The limitations which will be found after going through this study are given below:

1. This study is only for a normally distributed lead time.
2. Some assumptions are followed due to finding the outcome, but this has not been too much affected by those assumptions.
3. This research is only applicable for stock dependent items.

4 Model Formulation

Inventory control is important to maintain the right balance of stock in warehouses. In the simplest of terms, inventory control involves having greater oversight over one's stock. A business using the cycle inventory method might count different items at different rates, based on the level of turnover or demand for that particular item.

In figure 1. Lead time is not constant. Demand is constant up to re-order point. Then it becomes uncertain. That's why after re-order point R, various demand lines are shown. In this figure x-axis represents time and y-axis represents inventory on hand.

4.1 Assumptions

1. Demand is random during lead time.
2. Lead time is normally distributed.
3. Order quantity varies with time.
4. Demand is constant up-to reorder point.
5. A shortage occurs & penalty cost is applicable.

4.2 Notations

The following notations are used after reviewing several kinds of literature and considering some practical situations which are divided into parameters and variables

The model starts with the inventory consumption differential equation which is stock-dependent up to reorder point & stochastic according to *Rathod et.al.*. Thereafter,

$$\frac{dI(t)}{dt} = -\{D+\beta I(t)\}; 0 \le t \le t_k \qquad (1)$$

[Where negative sign indicate inventory reduce as demand serve]

=-x (Mean μ, Standard deviation σ) (2)

In order to solve I (t), we get from the above equation (1) by integrating both sides,

$$\int_0^t \frac{dI(t)}{D+\beta I(t)} = -\int_0^t dt$$

Let's,

$$D+\beta I = x$$

$$\beta dI(t) = dx$$

$$dI(t) = \frac{1}{\beta} dx$$

$$\int_0^t \frac{\frac{1}{\beta}}{x} dx = -\int_0^t dt$$

$$\frac{D+\beta I(t)}{D+\beta I(0)} = e^{-\beta t} \qquad (3)$$

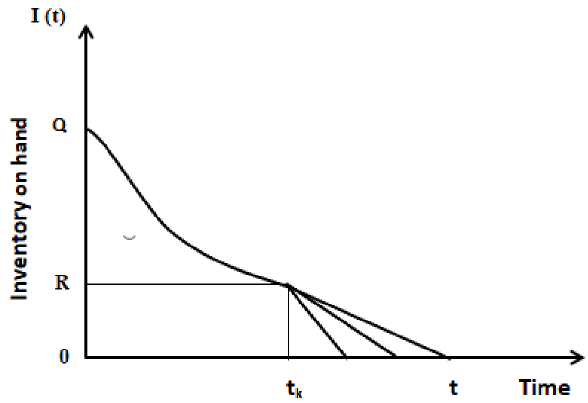

Fig.1. Inventory cycle during stochastic demand.

TABLE 1
Units for This Formulation

Symbol	Description
Parameters	
I(t)	Inventory on hand at time t
D	Demand Rate
A	Ordering Cost
β	Demand parameter indicating elasticity in relation to the inventory
P	Shortage/Penalty Cost
h_k	Holding Cost/Item
t	Time
T	Cycle Time
TC	Total Cost
μ	Mean of demand uncertainty
σ	Standard Deviation
Variables	
Q	Ordered Quantity
R	Re-order quantity

Using the boundary condition I (0) =Q we get,

$$I(t) = 1/\beta\left\{(D+\beta Q)e^{(-\beta t)} - D\right\} \qquad (4)$$

It's an instantaneous inventory. At t=t_k, and using the boundary condition I (t) =R in equation (4),

$$D+\beta R = (D+\beta Q)e^{(-\beta t_k)}$$

Where, Inventory I (t) = Reorder quantity(R).

$$t_k = \frac{1}{\beta} ln \frac{D+\beta Q}{D+\beta R} \quad (5)$$

This is an equation of time up to re-order point.
The mean holding cost is the summation of holding cost incurred up to reorder point and after reordering point
Now, holding cost (HC) is,

$$HC = h_k \int_0^{t_k} I(t)\,dt + h_K (R-\mu)$$

$$= h_k / \beta^2 (D+\beta Q)(1-e^{-\beta t_k}) - \frac{dt_k h_k}{\beta} + h_k (R-\mu) \quad (6)$$

The mean ordering cost per cycle is the division of ordering cost per order and mean cycle time.
Now, ordering cost=A/T

$$= \frac{A}{t_k + \tau}$$

Ordering cost= $\frac{A}{\frac{1}{\beta}\ln\left(\frac{D+\beta Q}{D+\beta R}\right)+\tau}$ (7)

According to the study of Leibnitz theorem expected number of unit stock out per cycle=n(R)

$$n(R) = \int_0^R (R-X)f(x)dx + \int_R^\infty (R-X)f(x)dx = L\delta(z)$$

Where L=Standard loss function and z= $\frac{x-\mu}{\delta}$

Shortage cost= $P\frac{n(R)}{T}$

$$= P\frac{n(R)}{\frac{1}{\beta} ln \frac{D+\beta Q}{D+\beta R} + \tau} \quad (8)$$

Finally, the total cost is the summation of holding cost, ordering cost and shortage cost.

$$TC= \frac{h_k}{\beta^2}(D+\beta Q)\left(1-e^{-\beta t_k}\right) - \frac{Dh_k t_k}{\beta} + h_k(R-\mu) + \frac{A}{\frac{1}{\beta}\ln\left(\frac{D+\beta Q}{D+\beta R}\right)+\tau} + P\frac{n(R)}{\frac{1}{\beta}ln\frac{D+\beta Q}{D+\beta R}+\tau}$$

$$= \frac{h_k}{\beta^2}(D+\beta Q)\left(1+\frac{D+\beta Q}{D+\beta R}\right) - \frac{Dh_k\frac{1}{\beta}}{\beta} ln\frac{D+\beta Q}{D+\beta R} + h_k(R-\mu) + \frac{A}{\frac{1}{\beta}\ln\left(\frac{D+\beta Q}{D+\beta R}\right)+\tau} + P\frac{n(R)}{\frac{1}{\beta}ln\frac{D+\beta Q}{D+\beta R}+\tau} \quad (9)$$

Now, total cost equation (9) is differentiated with respect to order quantity Q

$$\frac{h_k}{\beta^2}((D+\beta Q)\left(1+\frac{D+\beta Q}{D+\beta R}\right) - \frac{Dh_k\frac{1}{\beta}}{\beta}\ln\frac{D+\beta Q}{D+\beta R} + h_k(R-\mu) + \frac{A}{\frac{1}{\beta}ln\left(\frac{D+\beta Q}{D+\beta R}\right)+\tau} + \frac{P\,n(R)}{\frac{1}{\beta}ln\left(\frac{D+\beta Q}{D+\beta R}\right)+\tau} = 0$$

$$\frac{d(TC)}{dQ} = \frac{2h_k D}{\beta D+\beta^2 R} + \frac{2h_k \beta Q}{\beta D+\beta^2 R} + \frac{h_k Q}{\beta} - \frac{Dh_k}{\beta^2}\frac{\beta}{(D+\beta Q)} + 0 - A\left\{\frac{1}{\beta}ln(D+\beta Q) - \frac{1}{\beta}ln(D+\beta R) + \tau\right\}^{-2}$$
$$\times\left(\frac{1}{\beta}\frac{\beta}{D+\beta Q}\right) - P\,n(R)\left\{\frac{1}{\beta}ln(D+\beta Q) - \frac{1}{\beta}ln(D+\beta R)+\tau\right\}^{-2}\left(\frac{1}{\beta}\frac{\beta}{D+\beta Q}\right) = 0$$

$$\frac{d(TC)}{dQ} = \frac{h_k}{\beta} + \frac{2h_k(D+\beta Q)}{\beta(D+\beta R)} - \frac{Dh_k}{\beta(D+\beta Q)} - \frac{A}{(\frac{1}{\beta}ln\frac{D+\beta Q}{D+\beta R}+\tau)^2(D+\beta Q)}$$
$$-\frac{P\,n(R)}{\left(\frac{1}{\beta}ln\frac{D+\beta Q}{D+\beta R}+\tau\right)^2(D+\beta Q)} = 0 \quad (10)$$

Now, total cost is differentiated with respect to reordering quantity R,

$$\frac{h_k}{\beta^2}((D+\beta Q)\left(1+\frac{D+\beta Q}{D+\beta R}\right) - \frac{Dh_k\frac{1}{\beta}}{\beta}\ln\frac{D+\beta Q}{D+\beta R} + h_k(R-\mu) + \frac{A}{\frac{1}{\beta}ln\left(\frac{D+\beta Q}{D+\beta R}\right)+\tau} +$$
$$\frac{P\,n(R)}{\frac{1}{\beta}ln\left(\frac{D+\beta Q}{D+\beta R}\right)+\tau} = 0$$

$$\frac{d(TC)}{dR} = -\frac{(2h_k\beta DQ + h_k D^2 + h_k\beta^2 Q^2)}{(\beta^2 D+\beta^3 R)^2}\beta^3 + \frac{Dh_k}{\beta(D+\beta R)} + h_k + \frac{A}{\frac{1}{\beta}ln(\frac{D+\beta Q}{D+\beta R})+\tau)^2(D+\beta R)}$$
$$-\{-P(1-F(R)\}\left\{\frac{1}{\beta}ln(D+\beta Q) - \frac{1}{\beta}ln(D+\beta R)+\tau\right\}^{-2}(-\frac{1}{\beta}\frac{\beta}{(D+\beta R)}) = 0$$

[Here, F(R) = probability of not stocking out during lead-time]

$$\frac{d(TC)}{dR} = h_k - \frac{h_k\left(2D\beta Q + D^2 + \beta^2 Q^2\right)}{\left(\beta D^2 + \beta^3 R^2 + 2DR\beta^2\right)} + \frac{Dh_k}{\beta(D+\beta R)} + \frac{A}{(\frac{1}{\beta}ln\frac{D+\beta Q}{D+\beta R}+\tau)^2(D+\beta R)}$$
$$-\frac{P[1-F(R)]}{(\frac{1}{\beta}ln\frac{D+\beta Q}{D+\beta R}+\tau)^2(D+\beta R)} = 0 \quad (11)$$

From equation (9) & (10), optimum Q & R have been found for which total cost is minimum.

5 Model Implementation

In this research, we have developed a model of equations which can be used to reduce the total cost of inventory. The developed equations have been simulated in MATLAB. A code was generated by using those languages. The required data was collected from a renowned Bangladeshi food retail shop. The values are given below in the table2.

TABLE 2
COLLECTED DATA FOR MATLAB SIMULATION

Parameters	Values	Parameters	Values
Holding Cost (TK), h_k	1	Mean, μ	125
Ordering Cost (TK), A	100	Standard Deviation, Ϭ	5
Penalty Cost (TK), P	50	Cycle Time, T	5
Demand Rate, D	25	Demand Standard Deviation $Ϭ_z$	11

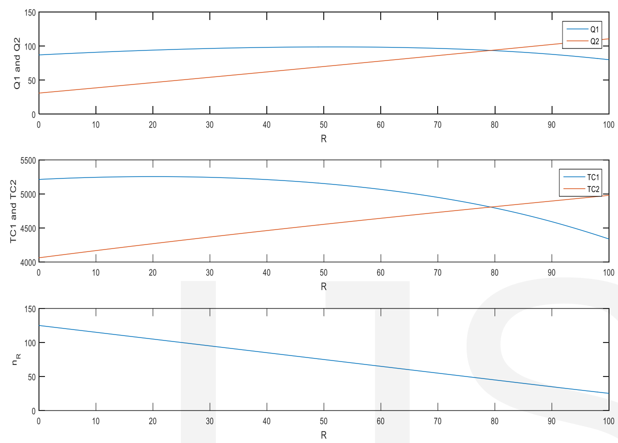

Fig. 2. Output of the MATLAB according to the values of table 2.

Some values were assumed for running the code. The assumed values are shown in table 2 and some others supportive values are reorder quantity's range= 0-100, demand elasticity, β= 0.1, tolerance= 1.

After putting these values into the MATLAB final equation, some values were found for:

1. Reorder quantity "R" for minimum total cost
2. Order quantity "Q"
3. Fill rate "b"
4. Total Cost "TC"

This is the primary result that was found. This does not express the optimum values. For the value limit of R= 0 to 100, Q_1 and Q_2 have been found and when the condition was $Q_1 = Q_2$ or , Q_1-Q_2 ≤ €,optimum R has been found .By using optimum Q and R ,optimum total cost has been found

TABLE 3
THE EFFECTS OF HOLDING COST

h_k	TC_{opt}	h_k	TC_{opt}
1.0	4800	1.6	6700
1.1	5200	1.7	7150
1.2	5600	1.8	7300
1.3	5850	1.9	7700
1.4	6200	2.0	7900
1.5	6600		

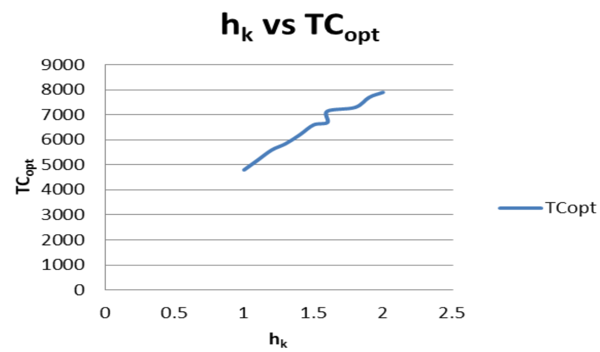

Fig. 3. Effects of Holding Cost on Total Cost

5.1 Sensitivity Analysis

By varying the values of h_k, B, P and A, a sensitivity analysis was performed which has given some required values for further calculations. These values helped to determine the optimum values for reorder point, fill rate, penalty cost and ordering cost. Optimum values were found from same sensitivity analysis

In table 3 the values of holding cost, h_k are varied from 1.0 to 2.0. Re-order quantity and ordering quantity are decreasing when h_k is increasing. For this reason the values of total cost are also decreasing as expectation. But the fill rate doesn't have a clear relationship. The values are changing to balance the total cost equation.

Here, figure 3 is the holding cost versus total cost graph. Holding cost is a part of total inventory cost. If holding cost is increased, total cost should increase too. This graph proves this commensurate relationship between total cost and holding cost. According to this diagram, 100% holding cost can increase around the total cost by about 40%.

By table 4, demand elasticity β has the same effect as holding cost. With the increment of β, re-order quantity, ordering quantity and total cost are decreasing. But here changes are visible in terms of fill rate, b. It is showing a downwards relationship with demand elasticity

TABLE 4
THE EFFECTS OF DEMAND ELASTICITY

β	TC_{opt}	β	TC_{opt}
0.10	4800	0.16	2550
0.11	4250	0.17	2350
0.12	3650	0.18	2300
0.13	3350	0.19	2150
0.14	3000	0.20	1950
0.15	2750		

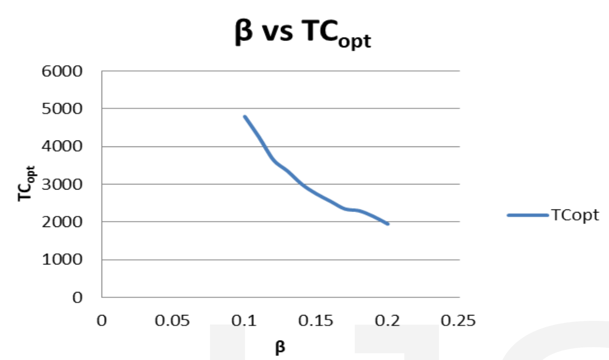

Fig. 4. Effects of Demand Elasticity on Total Cost

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The graph of figure 4 shows the effect of changing demand elasticity on total cost. Since β affects consumer demand which in turn determines the optimal values of the decision variables Q and R, increasing β will result in higher values of Q and R. Increasing R reduces the probability of stock-out and hence reduces total cost almost exponentially.

In the following table 5, all the re-order quantity, ordering quantity and fill rate has a proportional relationship with shortage cost, P. For this reason, total cost is showing the same relationship. In this figure 5, shortage cost has a noticeable impact on total cost. If shortage is increased, reorder point, order quantity also increase. Higher order quantity results in higher holding cost. But from the graph of h_k-Q, we have seen that if holding cost increases, order quantity decreases. This type of dichotomy causes a somewhat irregularity on total cost curve. But overall figure 5. is following an upward trend.

On the other hand, in table 6 re-order quantity is decreasing when ordering cost "A" is increasing. Ordering quantity is increasing when the values of A are being varied from 100-200. This the main reason for the same relation of TC with ordering cost in figure 6. But fill rate is not so much effected.

TABLE 5
THE EFFECTS OF SHORTAGE COST

P	TC_{opt}	P	TC_{opt}
50	4800	80	5200
55	4870	85	5300
60	4950	90	5350
65	5050	95	5360
70	5020	100	5300
75	5200		

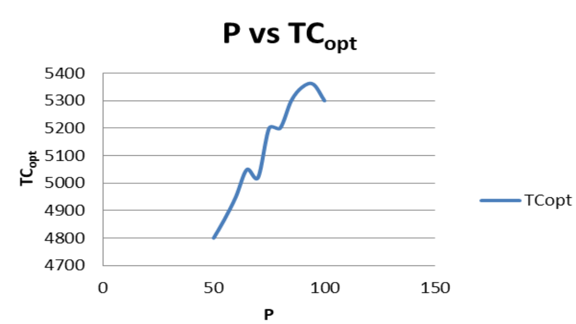

Fig. 5. Effects of Shortage Cost on Total Cost

TABLE 6
THE EFFECTS OF ORDERING COST

A	TC_{opt}	A	TC_{opt}
100	4800	160	4950
110	4750	170	4960
120	4800	180	5000
130	4850	190	5000
140	4870	200	5020
150	4900		

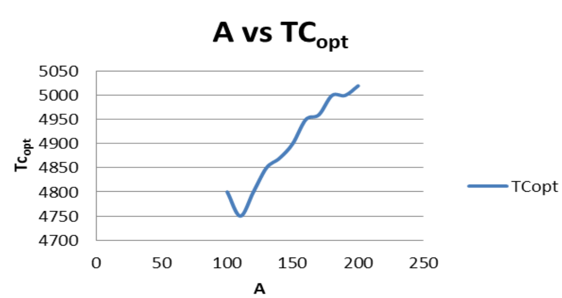

Fig. 6. Effects of Ordering Cost on Total Cost

From the above analysis, it can be concluded that the two most sensitive parameters for this particular inventory optimization problem are holding cost and demand elasticity.

6 Result & Comparison

The proposed model of inventory cost optimization has been successfully implemented in the previous section of this study. The objectives were obtained after the model implementations. The Total cost equation found is

$$TC= \frac{h_k}{\beta^2}\left(D+\beta Q\right)\left(1+\frac{D+\beta Q}{D+\beta R}\right)-\frac{Dh_k\frac{1}{\beta}}{\beta}ln\frac{D+\beta Q}{D+\beta R}$$
$$+h_k\left(R-\mu\right)+\frac{A}{\frac{1}{\beta}\ln\left(\frac{D+\beta Q}{D+\beta R}\right)+\tau}+P\frac{n(R)}{\frac{1}{\beta}ln\frac{D+\beta Q}{D+\beta R}+\tau} \quad (09)$$

Now, Authors have made decisions on the sensitive parameters according to the effect on total cost TC. The summery is given below in the table 7.

This table 7 is the base of reaching to the ultimate decision. From the table a decision can be made on the sensitive parameters. The effects of H_k and β are more on the total cost equation. When holding cost is increased by 10%, the effect is found maximum on the equation. With the increment of holding cost, total cost is increasing. The 2nd sensitive parameter is β. But this parameter shows the opposite effect. That means, when β has the highest value, total cost become lowest. Both of the two parameters have combined effect on total cost. To minimize the total inventory cost, holding cost should be low and β should be kept high as much as possible. Table 6.1 is helpful to make a decision that how much the holding cost and demand elasticity should be changed. The range of these two variables can be varied according to the expectation of an inventory controller to maintain the expected inventory cost.

7 Future Scope

Future researches can be done on this study.

1. To find the range of percentage of holding cost for obtaining the lowest inventory cost. This study doesn't show the specific range. This study can be a base research for finding that optimum range.
2. A research can be done to reduce the total supply chain cost by controlling the inventory cost for any specific multinational company. This research has a limitation that some values were assumed. But by collecting the accurate inventory data of any specific company, a reasonable supply chain cost equation can be achieved.
3. A research has a scope to minimize the inventory cost for certain demand and stochastic lead time. Lead time has kept constant during stochastic demand for this study. But a new research can show the effects of inventory cost on total cost of supply chain when demand is certain but lead time is uncertain.

We plan to work on these areas in future.

TABLE 7

Summary of the effects of various parameters

Parameters	Range	Starting value	Ending value	Type of effect
Holding Cost, h_k	01-02	4800	7900	Proportional
Demand Elasticity, β	0.1-0.2	4800	1950	Inversely Proportional
Penalty Cost, P	50-100	4800	5300	Proportional
Ordering Cost, A	100-200	4800	5020	Proportional

8 Conclusion

In this research, Q, R model was used to minimize the total inventory cost for stochastic demand when lead time is normally distributed. . Many authors have done many researches on this topic. But the contribution of this thesis is the development of a hybrid model of cost optimization. There is no study on an inventory hybrid model. This model is concerned with both certainty and uncertainty of demand. In the inventory cycle, previous studies were kept the same of either probabilistic or deterministic through-out the complete cycle. No combination of certain and stochastic demand. The important contribution of this thesis is the developed hybrid model. It will be helped to determine the cost when demand is a random variable after re-order point. The new model has been developed by considering the demand as deterministic up to re-order point. After re-order point customer demand has been considered as random variable.

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